Geometric Methods for Machine Vision

Abstract: This is a mini-course to be completed in twelve one hour lectures. The purpose of this course is to introduce problems from the general topic of machine vision. The selection of the topics are predominantly guided by my own research in the area of 'mathematics of vision', derived from robotics, computational neuroscience and biomechanics with applications ranging from motion and control of humans and machines. Although 'vision' has been used in multitude of engineering context as an extension of signal processing, this course emphasizes specifically on the role of dynamics and motion analysis from the point of view of observation and control. The subject makes a seamless contact with manifolds and geometry, which is the key point of this course.

Topics to be covered:

1. Classical introduction to monocular vision using geometry and dynamical systems.
2. Geometric problems in rotational mechanics and human gaze control.
3. Neural Signal Processing in Animal Vision.

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Prerequisite: It is anticipated that the audience for this course would be senior undergraduates and graduate students from engineering and mathematics. We will require knowledge of linear algebra, matrix theory and ordinary differential equations.

List of References:

[1]. B. K. Ghosh, *Systems Theory and Dynamic Control Problems in Vision*, Springer Verlag, Applied Sciences and Engg., Berlin, Heidelberg, 2016.

TOPIC: I

Lec 1: **Shape from Shading:**

Feature Based Description of Rigid Motion. Projective Geometric Description of Points, Lines and Planes. CCD cameras and projection models. Optical Flow description from Horn, Kanade and Kanatani.

Lec 2: **Shape Dynamics and Coupled Riccati Equation:**

Riccati's description of optical flow. Motion observed under perspective and orthographic projection, Coupled Riccati equation description of optical flow. Examples

Lec 3: **Perspective Systems Theory:**

Observability and parameter identification for linear systems in Euclidean and projective spaces. Orbits and group action on a manifold. Extension of the Popov Belevitch Hautus test of observability. Identification of motion parameters.

Lec 4: **Parameter Estimation using Observers and Filters:**

Kalman Filter and its extension, Nonlinear observer design, Application to range and shape estimation.

TOPIC: II

Lec 5: **Rotations, Quaternions and Mechanics:**

The space SO(3) of rotation matrices, A quaternion description of rotation in R^3. Various parametrizations of the space of unit quaternion. The space of rotation matrices as a Riemannian manifold. Euler Lagrange's equation describing rotational mechanics.

Lec 6: **Listing's and Donders' Laws for Eye and Head Movements:**

Eye and Head movement as a dynamics on SO(3) under constraints on the axes of rotation. Listing's and Donders' theorems. Geodesics on the constrained spaces LIST and DOND.

Lec 7: **The Control Problems: Potential and Optimal**

Eye and Head movement as a control system with externally applied torque as a control. Tracking of point targets and formulation as an optimal control problem, Associated two point boundary value problem. Solution using calculus of variation.

Lec 8: **Binocular Vision: Version and Vergence Eye Movements**

Listing's and Listing-like constraints for version and vergence eye movements. Optimal control problems in Binocular target acquisition and tracking. Some comments on spectral methods.

TOPIC: III

Lec 9: **What is a neuron?**

Neuron as a spike generating machine. Coupling of neurons using axons and dendrites. Mathematical models starting from the Hodgekin Huxley model and its simplifications. Discussion of the limit cycle behavior.

Lec 10: **What is a Computational Neuroscience?**

Population behavior of neural spike generators. Encoding and decoding functions and solving simple differential equations. Motor control using a population of computational neurons.

Lec 11: **A computational model of turtle retina.**

A turtle retina model using spatial distribution of retinal cells. Signal processing on the retinal response using principal component analysis. Modeling of the retinal response using a family of coupled point processes.

Lec 12: **A compartmental model of the turtle visual cortex.**

A detailed compartmental model of the turtle cortex generating cortical waves. Principal component analysis of waves in spatial and temporal domains, Detection of location and motion parameters using statistical detection Karhunen Loeve method.